# INTRINSIC IMAGE DECOMPOSITION BY HIERARCHICAL L<sub>0</sub> SPARSITY

*Xuecheng Nie*<sup>1,2</sup>, *Wei Feng*<sup>1,3,\*</sup>, *Liang Wan*<sup>2,3</sup>, *Haipeng Dai*<sup>1</sup>, *Chi-Man Pun*<sup>4</sup>

<sup>1</sup> School of Computer Science and Technology, Tianjin University, Tianjin, China
 <sup>2</sup> School of Computer Software, Tianjin University, Tianjin, China
 <sup>3</sup> Tianjin Key Lab for Advanced Signal Processing, Civil Aviation University of China, Tianjin, China

<sup>4</sup> Faculty of Science and Technology, University of Macau, Macau, China

## ABSTRACT

This paper presents a hierarchical approach to single image intrinsic decomposition based on non-local  $L_0$  sparsity. In contrast to previous studies using heuristic methods to welldefine the ill-posed problem, our approach is able to effectively construct sparse, non-local and multiscale reflectance dependencies in an unsupervised manner, thus is less dependent on the chromaticity feature and more accurately captures the global reflectance correlations. Besides, we impose homogenous smoothness prior and scale constraint in our model to further improve the decomposition accuracy. We formulate the decomposition as a quadratic minimization problem, which can be efficiently solved in closed form. Extensive experiments show that our approach can successfully extract the shading and reflectance components from a single image, and outperforms state-of-the-art methods on benchmark dataset. Besides, our approach can achieve comparable results with user-assisted methods on natural scenes.

*Index Terms*— Intrinsic image decomposition, hierarchical approach, non-local prior,  $L_0$  sparsity

# 1. INTRODUCTION

Intrinsic image decomposition targets at separating an input image into material-dependent component and lightingdependent component, known as reflectance and shading, respectively. It was originally proposed by Barrow and Tenenbaum [1] to describe the intrinsic characteristics of scenes. Since each component represents a different physical element, intrinsic image decomposition can benefit a lot of tasks both in computer graphics and computer vision, *e.g.* colorization and re-lighting [2], image segmentation [3], and object recognition. However, this problem still remains a challenging task due to its severe ill-posed nature: given an input image, the



**Fig. 1.** Single image intrinsic decomposition by hierarchical  $L_0$  sparsity. (a) shows some examples of non-local  $L_0$  sparse constraint at the original scale. (b) and (c) are the decomposed reflectance and shading components of our approach. (d) illustrates the generation process of hierarchical  $L_0$  sparse dependencies.

number of unknowns is twice the number of equations. To resolve the ambiguity, some early studies utilize images with different illumination conditions [4, 5], and achieve high quality results. But the strict input requirement limits the application of these multi-image based approaches. In recent years, intrinsic image decomposition from single image has drawn significant attentions.

**Relation to prior work.** Given a single image, previous studies have proposed different priors on reflectance and shading components. Retinex model [6] is the most widely used prior that analyzes local variations on image chromaticity. It assumes that small variations are caused by shading change and large variations are caused by reflectance change. This assumption is intuitively simple yet may not hold in images of real scenes, and binary classification is often unreliable. Training-based methods [7, 8] have been developed to attribute the image variations to shading or reflectance. However, the training convergence is input dependent and it is difficult to train all inclusive rules. Recently, Shen *et al.* [9] represented the reflectance value of one pixel by a weighted summation of its neighbors in a local window, and formulated intrinsic image decomposition as a minimization problem.

<sup>\*</sup> is the corresponding author. Email: wfeng@tju.edu.cn. This work is supported by NSFC (61100121, 61100122), the Program for New Century Excellent Talents in University (NCET-11-0365), the National Science and Technology Support Project (2013BAK01B01), and in part by Science and Technology Development Fund of Macau (008/2013/A1 and 034/2010/A2).

While the above approaches utilize local priors to resolve the inherent ambiguity, some recent studies show that nonlocal priors can help to reserve the global consistency and significantly improve the decomposition results. Shen and Yeo imposed a global sparsity prior of reflectance in [10], which assumes a small number of colors in the reflectance component. They formulated this prior as a total-variation cost on the set of reflectance values in the image. Zhao et al. [11] proposed a non-local reflectance constraint through texture analysis: if distant pixels have similar local texture structure, they are expected to have the same reflectance. This constraint is imposed by clustering pixels of the chromaticity image into groups. It is noticed that most previous methods depended on the chromaticity feature, which is not always reliable for images of real scenes, and employed heuristic ways to make the ill-posed decomposition well-defined.

In recent years,  $L_0$  sparsity has been used in many graphics and vision tasks. Xu *et al.* [12] developed  $L_0$  gradient minimization to implement image smoothing, which can globally control the number of non-zero gradients to approximate prominent image structures. Later, they extended the  $L_0$  gradient sparsity to solve the problem of image debluring [13], and achieved significant improvement both in efficiency and accuracy. Wang *et al.* [14] proposed a graph-cut approach to image segmentation by building an affinity graph based on  $L_0$  sparse representation of features. Experimental results on benchmark datasets show that their method can capture semantically meaningful regions and achieve competitive segmentation results compared to state-of-the-art techniques. The success of applying  $L_0$  sparsity in these recent tasks inspires us to utilize it in intrinsic image decomposition.

Our contribution. In this paper, we propose a hierarchical approach based on non-local  $L_0$  sparsity for intrinsic image decomposition. There are two major contributions of this work. First, we effectively construct sparse and non-local pairwise dependencies on reflectance component in an unsupervised manner. For each pixel of the given image, we construct a sparse representation by solving the  $L_0$  minimization problem, based on which global correlations are built to formulate a reliable prior on reflectance component. Second, the hierarchical approach makes intrinsic image decomposition much less dependent on the chromaticity feature. Our approach utilizes a coarse-to-fine process to propagate the correlations of reflectance from bottom to up image layers. Except the bottom layer, all the other layers use the combination of the decomposed reflectance propagated from the previous layer and chromaticity feature as the initial reflectance component. Experiments on benchmark dataset show that our approach helps to preserve the global consistency of the shading and reflectance component. Moreover, less dependence on chromaticity feature makes our approach more robust to images of real scenes. Comparisons with state-of-the-art techniques demonstrate the superior performance of our approach both in decomposition quality and perception.

## 2. OUR APPROACH

#### 2.1. Formulation

Let *I* represent the input image, *S* and *R* are its shading component and reflectance component, respectively. The intrinsic decomposition can be modeled as I = S + R in logarithmic space, where for simplicity *I*, *S* and *R* represent their log values. We define a new energy function to obtain the intrinsic images from a single image:

$$F(S,R) = f_s(S) + f_r(R) + f_a(S).$$
 (1)

Here,  $f_s(S)$ ,  $f_r(R)$  and  $f_a(S)$  formulate local homogenous smoothing constraint of shading, non-local  $L_0$  sparse constraint of reflectance and global scaling constraint.

Specifically, following the common Retinex constraint, the local homogenous smoothing constraint assumes that the change of shading between neighboring pixels should be small. That is to say, neighboring pixels with similar intensity values share similar shading values. We define  $f_s(S)$  as

$$f_s(S) = \sum_{i \leftrightarrow j} w_{ij}^S (S_i - S_j)^2, \qquad (2)$$

where  $i \leftrightarrow j$  denotes a pair of neighboring pixels *i* and *j*.  $S_i$  and  $S_j$  represent their shading values.  $w_{ij}^S$  measures the similarity between neighboring pixels *i* and *j*, given by:

$$w_{ij}^{S} = e^{-(Y_i - Y_j)^2 / \sigma_i^2}$$
(3)

where  $Y_i$  is the intensity value of pixel *i*, and  $\sigma_i^2$  is the intensity variance in a neighbourhood. We can then construct the shading similarity matrix  $W^S = \{w_{ij}^S\}$  for image *I*.

We formulate the non-local  $L_0$  sparse constraint of reflectance based on the assumption that a natural image can be well-defined by a small set of colors. Accordingly the reflectance of a given pixel can be represented sparsely by a small set of pixels in the image. We define  $f_r(R)$  as

$$f_r(R) = \sum_{i \sim j} w_{ij}^R (R_i - R_j)^2,$$
 (4)

where  $R_i$  is the reflectance of pixel *i*, and  $i \sim j$  denotes a pair of distant pixels. We formulate sparse representation of reflectance as a  $L_0$  minimization problem. By solving the sparse representation of reflectance component, we can construct global pairwise correlations, which can be referred as similarity metric  $w_{ij}^R$  of reflectance for distant pixel pair (i, j). The specific definition of  $w_{ij}^R$  will be presented in 2.2.

As pointed out in [11], there is a scale ambiguity between reflectance and shading components. If  $R^*$  and  $S^*$  are rational decomposition results,  $R^* + k$  and  $S^* - k$  are also rational for any scalar factor k. To solve this ambiguity, we add scaling constraint on shading component to ensure brightest pixel(s) to have unit shading value, which is defined as:

$$f_a(S) = \sum_{i \in \mathcal{B}} S_i^2.$$
(5)

where  $\mathcal{B}$  represents the set of brightest pixel(s) in image *I*.

## 2.2. Non-local L<sub>0</sub> sparse constraint of reflectance

For a given input image I with N pixels, we represent the feature of a pixel by concatenating the three channel reflectance values, which are initialized by chromaticity, at nearby pixels within a local window of size K. The feature set of all pixels is denoted as  $\mathcal{F} = \{X_i\}_{i=1}^N$ , where  $X_i$  is the feature vector of pixel i and its dimension is  $3K^2$ .

Based on the assumption mentioned in 2.1, the reflectance of a given pixel can be sparsely represented by the other pixels in the image. For each pixel i, we achieve the sparse representation by solving the following  $L_0$ -minimization problem:

$$\min_{\alpha_i} \|X_i - D_i \alpha_i\|_2^2 \quad s.t. \quad \|\alpha_i\|_0 \le \tau, \tag{6}$$

where  $D_i = [X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_N]$  is the sparse dictionary for pixel  $i, \alpha_i \in \mathbb{R}^{N-1}$  is the coefficient vector for sparse representation of  $X_i$  over  $D_i$ . The  $L_0$  norm  $\|\alpha_i\|_0$  returns the number of non-zero coefficients in  $\alpha_i$ , and the parameter  $\tau$  controls the representation sparsity. The set of coefficient vectors for all pixels is denoted as  $\alpha = \{\alpha_i\}_{i=1}^N$ . This  $L_0$  minimization problem can be solved by Orthogonal Matching Pursuit (OMP) algorithm [15], which achieves the best linear representation of  $X_i$  on dictionary  $D_i$ .

The non-zero coefficients in  $\alpha_i$  provide correlations of pixel *i* with other pixels, based on which various similarity metrics have been defined [14, 16]. In this paper, we use the Normalized-Residual to measure the similarity of reflectance for pixel pair (i, j), which is defined as following:

$$w_{ij}^{R} = \frac{\|X_i - D_i \alpha_i^{j}\|_2^2}{\|X_i\|_2^2},$$
(7)

where

$$\alpha_i^j(l) = \begin{cases} 0, & \text{if } l = j, l = 1, \dots, N-1, \\ \alpha_i(l), & \text{otherwise.} \end{cases}$$
(8)

By this definition, the more similar pixel *i* and pixel *j* are, the larger  $\alpha_i(j)$  will be, which results in larger  $w_{ij}^R$ . Similar to W<sup>S</sup>, we construct the reflectance similarity matrix W<sup>R</sup> =  $\{w_{ij}^R\}$  for all pixel pairs of image *I*.

This  $L_0$  sparse representation for reflectance component can help to achieve the non-local correlation for all pixel pairs in the image, which encourages the global consistency to intrinsic image decomposition. Despite the rationality and effectiveness of  $L_0$  sparse representation, there are two problems of working on the original image: first, the sparse dictionary is always very large which results that the solving process is time-consuming; second, the initialization of reflectance is absolutely dependent on the chromaticity feature. To overcome these two problems, we propose a hierarchical model utilizing a coarse-to-fine process, as will be specified in the next section.

#### **2.3.** Hierarchical L<sub>0</sub> sparse constraint of reflectance

For image I, we build an image pyramid  $\mathcal{P}_I = \{I^k\}_{k=1}^M$ , given the downsampling ratio  $\kappa$  and the number of hierarchy M, where  $I^k$  represents the k-th image layer in  $\mathcal{P}_I$  from up to bottom. Here, the bottom layer is right the original image I. Similarly, we construct a chromaticity pyramid  $\mathcal{P}_C = \{C^k\}_{k=1}^M$ , where  $C^k$  is the corresponding chromaticity image of  $I^k$ .

For image layer  $I^1$ , we initialize the reflectance component  $\hat{R}^1$  with chromaticity  $C^1$ . Based on  $\hat{R}^1$ , we build the feature set  $\mathcal{F}^1 = \{X_i^1\}_{i=1}^{N^1}$  and the sparse dictionary  $\mathcal{D}^1 = \{D_i^1\}_{i=1}^{N^1}$  for all pixels of  $I^1$ , where  $N^1$  represents the number of pixels in  $I^1$ . By solving the set of coefficient vectors  $\alpha^1 = \{\alpha_i^1\}_{i=1}^{N^1}$  using Eq.(6), we can construct the reflectance similarity matrix  $W^{R^1}$ . Besides, the shading similarity matrix  $W^{S^1}$  can also be constructed as mentioned in 2.1. Considering non-local  $L_0$  sparse constraint, local homogenous smoothing constraint and scaling constraint, we decompose  $I^1$  into the reflectance component  $R^1$  and shading component  $S^1$  based on Eq.(1). The solving process will be illustrated in detail in 2.4.

For image layer  $I^k(1 < k \leq M)$ , we initialize the reflectance component  $\widehat{R}^k$  as following:

$$\widehat{R}^k = \lambda C^k + (1 - \lambda) R^{k-1}_{\uparrow}, \qquad (9)$$

where  $R^{k-1}_{\uparrow}$  represents the upsampling reflectance component of  $R^{k-1}$  with ratio  $\kappa$ . Based on  $\widehat{R}^k$ , we build feature set  $\mathcal{F}^k$  for all pixels of  $I^k$ . In order to reduce the dictionary size effectively, we construct sparse dictionary  $\mathcal{D}^k$  in a manner different from  $\mathcal{D}^1$ . For each pixel j in image  $I^k$ , denoted as  $I_j^k$ , we find the corresponding pixel  $I_{j*}^{k-1}$  in  $I^{k-1}$  by downsampling (green point in  $I^{k-1}$  in Figure 1(d)). Then, we build the set of correlated pixels of  $I_{j*}^{k-1}$  by identifying the nonzero coefficients in  $\alpha_{j*}^{k-1}$ . Denote this correlated pixel set as  $\mathcal{G}_{j*}^{k-1}$  (red points in  $I^{k-1}$  in Figure 1(d)). For each pixel in  $\mathcal{G}_{i*}^{k-1}$ , we find the corresponding local patch with a window size of  $\kappa$  in image  $I^k$  by upsampling, and the set of all these corresponding local patches is denoted as  $\mathcal{H}_{i}^{k}$  (blue patches in  $I^k$  in Figure 1(d)). We use all the pixels in  $\mathcal{H}_j^k$  to construct the sparse representation dictionary  $D_j^k$  for  $I_j^k$ . Then, we solve the coefficient vector set  $\alpha^k$  for  $\mathcal{F}^k$  over  $\mathcal{D}^k$  and build the reflectance similarity matrix  $W^{R^k}$ . Combining the shading similarity matrix  $W^{S^k}$ , we decompose image  $I^k$  into the shading component  $S^k$  and reflectance component  $R^k$ .

By this hierarchical model, the size of sparse dictionary can be effectively reduced, and the decomposition algorithm can be significantly speeded up. Moreover, the reflectance component decomposed at low layer can refine the initialization of reflectance component of high layer, which makes the algorithm more robust to images of real scenes and helps improve the decomposition quality.

## Algorithm 1 Hierarchical Intrinsic decomposition

**Input:** image *I*, down/up-sampling ratio  $\kappa$ , hierarchy number *M*, sparsity  $\tau$ , alpha-blending weight  $\lambda$ . Initialization:  $\mathcal{P}_I = \{I^k\}_{k=1}^M$  and  $\mathcal{P}_C = \{C^k\}_{k=1}^M$ ; **for** k = 1 to *M* **do if** k is equal to 1 **then**   $\hat{R}^k \leftarrow C^k$ ; **else**   $\hat{R}^k \leftarrow \lambda C^k + (1 - \lambda) R^{k-1}_{\uparrow}$ ; **end if** Get  $\mathcal{F}^k$  and  $\mathcal{D}^k$  based on  $\hat{R}^k$ ; Get  $\alpha^k$  by solving Eq.(6); Get  $W^{R^k}$  using Eq.(7), get  $W^{S^k}$  using Eq.(3); Get  $S^k$  using Eq.(11)-(14), get  $R^k = I^k - S^k$ ; **end for**   $R \leftarrow R^M, S \leftarrow S^M$ ; **Output:** Reflectance component *R*, shade component *S*;

## 2.4. A closed-form solver

Since we can represent reflectance component R by I and S, i.e. R = I - S, the objective function F(S, R) can be simplified as following:

$$F(S) = \sum_{i \leftrightarrow j} w_{ij}^S (S_i - S_j)^2 + \sum_{i \sim j} w_{ij}^R (\Delta I_{ij} - S_i + S_j)^2 + \sum_{i \in \mathcal{B}} S_i^2$$
(10)

where  $\Delta I_{ij} = I_i - I_j$ . Obviously F(S) is a quadratic function with respect to shading component S. We can represent this function in a standard quadratic form as:  $\frac{1}{2}\mathbf{s}^{\top}\mathbf{A}\mathbf{s} - \mathbf{b}^{\top}\mathbf{s} + c$ . Through mathematical deduction, **A** has a form as follows:

$$\mathbf{A} = 4L(\mathbf{W}^{S*}) + 4L(\mathbf{W}^{R*}) + 2B.$$
(11)

Here,  $W^{S*} = \frac{1}{2}(W^S + W^{S^{\top}})$  is a symmetric matrix, where  $W^{S^{\top}}$  is the transposition of  $W^S$ .  $W^{R*}$  is defined in a similar way.  $L(W^{S*})$  and  $L(W^{R*})$  compute the Laplacian matrix from  $W^{S*}$  and  $W^{R*}$ , respectively. B is a diagonal matrix with  $B_{ii} = 1$  if  $S_i \in \mathcal{B}$ , otherwise,  $B_{ii} = 0$ .

As we know, the Laplacian matrix is semi-positive defined, so are  $L(W^{S*})$  and  $L(W^{R*})$ . Matrix B, which has non-negative diagonal elements only, is also semi-positive. Therefore, their linear combination **A** is a semi-positive matrix. The vector **b** is given by

$$\mathbf{b}(i) = \sum_{j}^{N} w_{ji}^{R} \Delta I_{ji} + \sum_{j}^{N} w_{ij}^{R} \Delta I_{ij}.$$
 (12)

c is a constant, which is:

$$c = \sum_{i}^{N} \sum_{j}^{N} w_{ij}^{R} \Delta I_{ij}^{2}.$$
(13)

It is well-known that if A is a symmetric and positive-defined matrix, the quadratic function has a unique global minimum

which is the solution of the linear system:

$$\mathbf{As} = \mathbf{b}.\tag{14}$$

This yields a closed-form solution to the intrinsic image decomposition problem defined in Eq.(10), which can be effectively solved by conjugate gradient algorithm. The complete algorithm of our approach is summarized in Algorithm 1.

#### 3. EXPERIMENTAL RESULTS

We first test our approach on the benchmark MIT dataset provided by [19]. The MIT dataset contains the ground truth of intrinsic image decompositions for three categories, including artificially painted surfaces, printed objects, and toy animals. We quantitatively evaluate the decomposition quality by computing the Local Mean Squared Error (LMSE). In our experiments, we compare our approach with two state-of-the-art algorithms for *automatic* intrinsic image decomposition from a single image: the conventional color Retinex [17], which is reported to have the best performance among existing methods that use local constraints, and the closed form solution using texture analysis [11] which is one of the latest works adopting sparse global constraints.

We display the decomposition results of one example of each image category in Table 1. Here, GT denotes the ground truth, CR denotes the color retinex algorithm, and CFS denotes the closed form solution [11]. We can see that both CFS and our method effectively separate the reflectance from the lighting in both "panther" and "turtle" examples, while CR fails to distinguish the two components. In "cup1", our shading result has no pattern, quite similar to that of the ground truth. Obviously, our approach outperforms other algorithms both in decomposition accuracy and perception.

We quantitatively evaluate the images in the MIT dataset and the results are shown in Table 2. Among the test images, our approach yields the lowest scores in 10 examples. We also compute the average LMSE score, and get 0.021 for our approach, lower than CR with 0.030 and CFS with 0.025. In addition, our approach can achieve high performance on some cases that chromaticity feature does not work well, *e.g.* "turtle", "frog2", and "teabag1". This shows that our algorithm is less dependent on the chromaticity feature, which can benefit the intrinsic image decomposition.

Besides the benchmark MIT dataset, we also compare our approach on natural images with two state-of-the-art userassisted methods [9, 18], which exploit three kinds of interactions, i.e. reflectance-constant scribbles, shading-constant scribbles and fixed-illumination scribbles, as extra constraints to well-define the intrinsic decomposition problem. As shown in Figure 2, our approach can successfully preserve the global consistency of shading and reflectance components for natural scenes, and is able to *automatically* achieve comparable decomposition results with the interactive methods [9, 18].

Image	s	panther					cup1			turtle			
GT				***	- Contraction				7				80
CR [17								9			8		P
		LMSE = 0.011					LMSE = 0.007			LMSE = 0.069			
CFS [1	1]		LMSE	= 0.006			LMSI	E = 0.010	7		LWSE =	= 0.037	20
Ours		-				•			7		20		
LMSE = 0.003							LMSE = 0.003			LMSE = 0.023			
Table 2. Decomposition results comparison of different approaches on all images of MIT dataset													
	box	cup2	deer	dinosaur	frog1	frog	2 paper1	paper2	racco	on squirrel	sun	teabag1	teabag2
CR	0.013	0.011	0.041	0.035	0.066	0.07	1 0.004	0.004	0.01	5 0.072	0.003	0.041	0.023
CFS	0.005	0.005	0.045	0.026	0.051	0.06	9 0.008	0.005	0.004	4 0.074	0.002	0.042	0.017
Ours	-0.007	0.004	0.042	0.028	0.050	0.04	6 0.002	0.005	(0.00)	4 0.073	0.003	0.020	0.026

Table 1. Decomposition results comparison of different approaches on three images of the MIT dataset

### 4. CONCLUSIONS

In this paper, we propose a hierarchical model to solve the intrinsic image decomposition problem. Our core is a novel non-local  $L_0$  sparse prior to preserve the global consistency. It is based on the assumption that the reflectance value of a pixel can be sparsely represented by other pixels in the image. Different from previous studies using heuristic ways to make the decomposition problem well-defined, our approach can construct non-local pairwise dependencies automatically, moreover, it is less dependent on chromaticity feature. Combining homogenous smoothing prior on shading component and scaling prior, we formulate the intrinsic image decomposition problem as the minimization of a quadratic function, which can be solved in closed form with the standard conjugate gradient algorithm. Evaluations on benchmark MIT

dataset show that our approach outperforms state-of-the-art techniques both in decomposition accuracy and perception. In addition, our approach can automatically achieve comparative results with user-assisted approaches on natural scenes.

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**Fig. 2.** Intrinsic decomposition for natural images. (a) are the original images. (b) and (c) are interactive decomposition results of [9]. (d) and (e) are interactive decomposition results of [18]. The results of (b)–(e) were generated using the same (reasonable) set of user scribbles. (f) and (g) are the decomposition results of our approach without any user-assistance.

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